INTERNATIONAL BACCALAUREATE (IB)
MATH STANDARD LEVEL, YEARS 1 AND 2
Grades 11, 12

Unit of Credit: 1 Year for Year 1 and 1 Year for Year 2

Pre-requisite: Algebra 2 for Year 1
IB Math Standard Level Year 1 for Year 2

Course Overview:
This course is aimed at students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

The course focuses on introducing important mathematical concepts through the development of mathematical techniques. The intention is to introduce students to these concepts in a comprehensible and coherent way, rather than insisting on mathematical rigor. Students should, wherever possible, apply the mathematical knowledge they have acquired to solve realistic problems set in an appropriate context.

The internally assessed component of this course, the portfolio, offers students a framework for developing independence in their mathematical learning by engaging in mathematical investigation and mathematical modeling. Students are provided with opportunities to take a considered approach to these activities and to explore different ways of approaching a problem. The portfolio also allows students to work without the time constraints of a written examination and to develop the skills they need for communicating mathematical ideas.

This course does not have the depth found in the IB Mathematics Higher Level course. Students wishing to study subjects with a high degree of mathematical content should therefore opt for the Mathematics Higher Level course rather than a Mathematics Standard Level course.

Aims
1. To appreciate the multicultural and historical perspectives of all group five courses.
2. To enjoy the courses and develop an appreciation of the elegance, power, and usefulness of the subjects.
3. To develop logical, critical and creative thinking.
4. To develop an understanding of the principles and nature of the subject.
5. To employ and refine their powers of abstraction and generalization.
6. To develop patience and persistence in problem solving.
7. To appreciate the consequences arising from technological developments.
8. To transfer skills to alternative situations and to future developments.
9. To communicate clearly and confidently in a variety of contexts.

Topic 1—Number and Algebra

The aim of this section is to introduce students to some basic algebraic concepts and applications.

- Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series.
- Sigma notation.
- Applications
- Elementary treatment of exponents and logarithms.
- Laws of exponents; laws of logarithms.
- Change of base.
- The binomial theorem: expansion of $(a + b)n$, $n \in$N.
- Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$
- **Not required:** formal treatment of permutations and formula for ${}_nP_r$

Topic 2—Functions and equations

The aims of this topic are to explore the notion of a function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic, rather than elaborate analytical techniques. On examination papers, questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus, and students may need to choose the appropriate viewing window. For those functions explicitly mentioned, questions may also be set on composition of these functions with the linear function $y = ax + b$

- Concept of function $f: x \mapsto f(x)$
- Domain, range; image (value).
- Composite functions.
- **Not required:** Domain restriction.
- Identity function. Inverse function f^{-1} .
- The graph of a function; its equation $y = f(x)$.
- Function graphing skills.
- Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range.
- Use of technology to graph a variety of functions, including ones not specifically mentioned.
- The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$
- Transformations of graphs.
- Translations: $y = f(x) + b$; $y = f(x - a)$.
- Reflections (in both axes): $y = -f(x)$; $y = f(-x)$
- Vertical stretch with scale factor p: $y = pf(x)$
- Stretch in the x-direction with scale factor $\frac{1}{q}$: $y = f(qx)$.
- Composite transformations. The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry.
- The form $x \mapsto a(x - p)(x - q)$, x-intercepts (p, 0) and (q, 0).
- The form $x \mapsto a(x - h)^2 + k$, vertex (h, k) .

- The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.
- The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph.
- Vertical and horizontal asymptotes.
- Exponential functions and their graphs: $x \mapsto a^x, a > 0, x \mapsto e^x$
- Logarithmic functions and their graphs: $x \mapsto log_a x,\ x > 0$, $x \mapsto \ln x$, $x > 0$.
- Relationships between these functions:
- $a^x = e^{xlna}$; $log_a a^x = x, a^{log_a x} = x, x > 0.$
- Solving equations, both graphically and analytically.
- Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.
- Solving $ax^2 + bx + c = 0, a \neq 0$. The quadratic formula.
- The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.
- Solving exponential equations.
- Applications of graphing skills and solving equations that relate to real-life situations.

Topic 3-Circular functions and trigonometry

The aims of this topic are to explore the circular functions and to solve problems using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated.

- The circle: radian measure of angles; length of an arc; area of a sector.
- Definition of $cos\theta$ and $sin\theta$ in terms of the unit circle.
- Definition of $tan\theta$ as $\frac{sin\theta}{cos\theta}$.
- Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.
- The Pythagorean identity $cos^2\theta + sin^2\theta = 1$.
- Double angle identities for sine and cosine.
- Relationship between trigonometric ratios.
- The circular functions $sin\ x$, $cos\ x$ and $tan\ x$: their domains and ranges; amplitude, their periodic nature; and their graphs.
- Composite functions of the form $f(x) = asin(b(x + c)) + d.$
- Transformations
- Applications
- Solving trigonometric equations in a finite interval, both graphically and analytically.
- Equations leading to quadratic equations in sin *x*, cos *x* or tan *x* .
- **Not required:** the general solution of trigonometric equations.
- Solution of triangles.
- The cosine rule.
- The sine rule, including the ambiguous case.
- Area of a triangle, $\frac{1}{2}absinC$.
- Applications.

Topic 4—Vectors

The aim of this topic is to provide an elementary introduction to vectors, including both algebraic and geometric approaches. The use of dynamic geometry software is extremely helpful to visualize situations in three dimensions.

- Vectors as displacements in the plane and in three dimensions
- Components of a vector; column representation $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1 i + v_2 j + v_3 k$.
- Algebraic and geometric approaches to the following:
  - the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;
  - multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors;
  - magnitude of a vector, $\boldsymbol{v}$;
  - unit vectors; base vectors; $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$;
  - position vectors $\overrightarrow{OA} = a$;
  - $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = b - a$.
- The scalar product of two vectors.
- Perpendicular vectors; parallel vectors.
- The angle between two vectors.
- Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.
- The angle between two lines.
- Distinguishing between coincident and parallel lines.
- Finding the point of intersection of two lines.
- Determining whether two lines intersect.
- Expected value.
- Inverse normal calculations.
- Not required:
  - Transformation of any normal variable to the standardized normal.
- Bivariate data: the concept of correlation.
- Scatter diagrams; line of best fit, by eye, passing through the mean point.

Topic 5—Statistics and Probability

The aim of this topic is to introduce basic concepts. It is expected that most of the calculations required will be done using technology, but explanations of calculations by hand may enhance understanding. The emphasis is on understanding and interpreting the results obtained, in context. Statistical tables will no longer be allowed in examinations. While many of the calculations required in examinations are estimates, it is likely that the command terms "write down", "find" and "calculate" will be used.

- Concepts of population, sample, random sample, discrete and continuous data.
- Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals; box-and-whisker plots; outliers.
- Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class.
- **Not required:** frequency density histograms.
- Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles, percentiles.
- Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data.

- Applications.
- Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.
- Linear correlation of bivariate data.
- Pearson's product–moment correlation coefficient *r*.
- Scatter diagrams; lines of best fit.
- Equation of the regression line of *y* on *x*. Use of the equation for prediction purposes. Mathematical and contextual interpretation.
- **Not required:** the coefficient of determination *R*2.
- Concepts of trial, outcome, equally likely outcomes, sample space (*U*) and event.
- The probability of an event *A* is $P(A) = \frac{n(A)}{n(U)}$.
- The complementary events *A* and *A*'(not *A*).
- Use of Venn diagrams, tree diagrams and tables of outcomes.
- Combined events, $P(A \cup B)$.
- Mutually exclusive events: $P(A \cap B) = 0$.
- Conditional probability; the definition
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$.
- Independent events; the definition P(*A*| *B*) = P(*A*) = P(*A*| *B*') .
- Probabilities with and without replacement.
- Concept of discrete random variables and their probability distributions.
- Expected value (mean), E(*X*) for discrete data.
- Applications.
- Binomial distribution.
- Mean and variance of the binomial distribution.
- **Not required:** formal proof of mean and variance.
- Normal distributions and curves.
- Standardization of normal variables (*z*-values, *z*-scores).
- Properties of the normal distribution.

Topic 5—Calculus

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their applications

- Informal ideas of limit and convergence.
- Limit notation.
- Definition of derivative from first principles as $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
- Derivative interpreted as gradient function and as rate of change.
- Tangents and normals, and their equations.
- **Not required:** analytic methods of calculating limits.
- Derivative of $x^n (n \in \mathbb{Q}), \sin x, \cos x, \tan x, e^x, \ln x$
- Differentiation of a sum and a real multiple of these functions.
- The chain rule for composite functions. The product and quotient rules.
- The second derivative.
- Extension to higher derivatives.

Year One

Introduction to the Graphical Display Calculator

- Arithmetic calculations
- Graphing functions
- Common buttons
- Data lists

Number and Algebra

- Sets of numbers; natural, integers, rational and real
- Approximation, decimal places, significant figures, percentage error, estimation
- Operations with scientific notation
- Metric system
- Arithmetic sequences and series
- Geometric sequences and series
- Solution of linear equations with GDC
- Solutions of quadratic equations by factorization and GDC

Functions

- Concepts of functions as a mapping, domain, range, mapping diagrams
- Linear functions and their graphs
- Quadratic functions, axis of symmetry, vertex, intercepts

Sets, Logic and Probability

- Basic concepts of set theory; subsets, intersection, union, complement
- Venn diagrams and simple applications
- Sample space and complementary events
- Equally likely events, probability of an event A given by $P(A) = \frac{n(A)}{n(U)}$ probability of a complementary event.
- Venn diagrams, tree diagrams, table of outcomes
- Laws of probability, combined events, mutually exclusive events, independent events, conditional probability

Geometry and Trigonometry

- Coordinates in two dimensions, points, lines, planes, distance formula
- Equations of line in two dimensions, gradient, intercepts parallel and perpendicular lines
- Right-angle trigonometry, trigonometric ratios
- Sine Rules, cosine rule, area of triangle, constructions
- Three dimensional geometry, cubes, prisms, pyramids, cylinders, spheres, cones

Statistics

- Classification of data as discrete or continuous
- Frequency tables and polygons
- Histograms stem and leaf diagrams, boundaries
- Cumulative frequency tables and graphs, box and whisker plots, percentiles, quartiles

- Measures of central tendency, mean, median, mode
- Measures of dispersion, range, interquartile range, standard deviation

Financial Mathematics

- Currency conversions
- Simple interest
- Compound interest, depreciation
- Construction and use of tables, loan and repayment schemes, investment and saving schemes, inflation

Year Two

Sets, Logic and Probability (continued)

- Concepts of symbolic logic, definition of proposition, notation
- Truth tables
- Definition of implication, converse, inverse and contrapositive
- Functions (continued)
- Exponential functions, growth and decay, asymptotic behavior
- Sine and cosine functions, amplitude, period
- Accurate graph drawing
- Use of GDC to sketch and analyze new functions. Statistics (continued)
- Scatter plots, line of best fit, bivariate data, Pearson's product-moment correlation coefficient, interpretation of correlations
- The regression line for y on x, use of regression line for predictions
- The Chi-Square test for independence, formulation of null and alternative hypothesis, significance levels, contingency tables, expected frequencies, degrees of freedom

Differential Calculus

- Gradient of two points on the graph of a function, behavior of the gradient as one point approaches other, tangent to a curve
- Derivatives of 1-variable monomials and polynomials
- Gradients of curves for given values of x, values of x where f '(x) is given, equations of the tangent at a given point
- Increasing and decreasing functions, graphical interpretation of derivatives
- Values of x where the gradient is zero, local maxima and minima

Project

- This is a significant piece of written work for which the student undertakes personal research on a mathematical project of their choice. This project, which is undertaken in the first semester of the second year, contributes the internal part of their IB math assessment.

Syllabus Review and Exam Preparation

- At the conclusion of the course in the final semester of the second year, students undertake a comprehensive review of the course material alongside preparation and practice for examinations.